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WAVE PROPAGATION IN CRUCIFORM ROD SYSTEMS

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It is often important to know the kind of vibrations of structural elements in vibration diagnostics problems of elastic structures. Transformation of the kind of vibrations is observed during propagation of vibrations in complex bifurcated structures and an element remote from the vibrations source can perform vibrations different from those which the source supplies.

Rods and plates are typical elements of elastic structures – consequently, a great deal of attention is usually paid to the study of vibrations propagating in rod and plate structures [1-3]. However, as a rule, rods and plates are studied in engineering theory approximations, which naturally constrains the frequency range of applicability of these models.

Theoretical and experimental investigations of the generation of longitudinal and bending waves through a cruciform connection of rods are carried out in this paper. The computation is performed by a nonclassical rod model [4]. This permits studying the wave processes in an elastic structure not only at low frequencies but also in the frequency range for which the lengths of the propagated waves become commensurate with the rod transverse dimensions.

1. A stiff cruciform connection of four rods is considered (Fig. 1). The propagation of longitudinal and bending waves in each of the rods is described by equations of the refined theory [4]

$$\rho_j S_j \frac{\partial^2 u_j}{\partial t^2} - E_j S_j \frac{\partial^2 u_j}{\partial x_j^2} - \rho_j \nu_j^2 I_{0j} \frac{\partial^4 u_j}{\partial x_j^2 \partial t^2} = 0,$$

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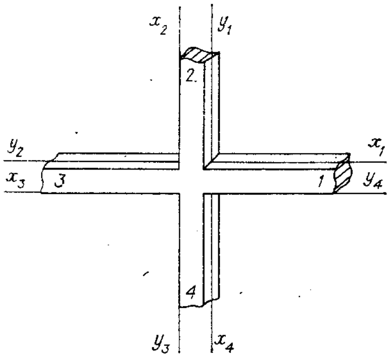

Fig. 1

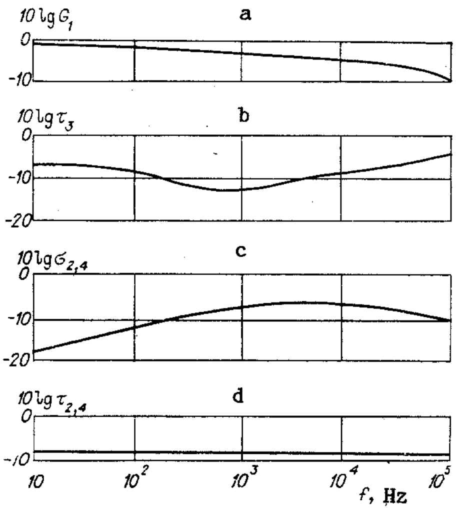

Fig. 2

$$\rho_j S_j \frac{\partial^2 w_j}{\partial t^2} + E_j I_j \frac{\partial^4 w_j}{\partial x_j^4} - \rho_j I_j \frac{\partial^4 w_j}{\partial x_j^2 \partial t^2} = 0,$$

where j = 1, ..., 4 is the rod number, u(x, t), w(x, t) are the longitudinal and transverse displacements of the rod particles, ρ, S, E are the volume density, the area of the transverse sections, and Young's modulus, and I, I_0 are the axial and polar moments of inertia.

In this approximation the longitudinal waves possess a weak high-frequency dispersion described by the expression

$$q_j = \pm \frac{w}{c_j} \left(1 - \frac{\nu_j^2 R_j^2}{c_j^2} \omega^2 \right)^{-1/2}.$$

The dispersion of the bending waves is determined by the relationship

$$k_j = \pm \left[\frac{\omega^2}{2c_j^2} \pm \left(\frac{\omega^4}{4c_j^4} + \frac{\omega^2}{c_j^2 r_j^2} \right)^{1/2} \right]^{1/2}.$$

Here ω is the circular frequency, q_j, k_j are the longitudinal and bending wave numbers, $c = (E/\rho)^{1/2}$ is the rod velocity, $r = (I/S)^{1/2}$, $R = (I_0/S)^{1/2}$ are the axial and polar radii and inertias.

The following boundary conditions

$$w_1 - u_4 = 0,\ w_4 - u_3 = 0,\ w_2 + u_3 = 0,$$
$$u_1 + u_3 = 0,\ w_3 + u_4 = 0,\ u_2 + u_4 = 0$$

the displacement continuities

$$\frac{\partial w_1}{\partial x_1} - \frac{\partial w_2}{\partial x_2} = 0, \quad \frac{\partial w_1}{\partial x_1} - \frac{\partial w_3}{\partial x_3} = 0, \quad \frac{\partial w_1}{\partial x_1} - \frac{\partial w_4}{\partial x_4} = 0$$

should be satisfied at the site of the rod connections (x_j = 0,) as should the continuities of the angle of rotation of the transverse sections during bending

$$E_1 I_1 \frac{\partial^3 w_1}{\partial x_1^3} - \rho_1 I_1 \frac{\partial^3 w_1}{\partial x_1 \partial t^2} + E_2 S_2 \frac{\partial u_2}{\partial x_2} - \rho_2 \nu_2^2 I_{02} \frac{\partial^3 u_2}{\partial x_2 \partial t^2} - E_3 I_3 \frac{\partial^3 w_3}{\partial x_3^3} +$$
$$+ \rho_3 I_3 \frac{\partial^3 w_3}{\partial x_3 \partial t^2} - E_4 S_4 \frac{\partial u_4}{\partial x_4} + \rho_4 \nu_4^2 I_{04} \frac{\partial^3 u_4}{\partial x_4 \partial t^2} = 0,$$

$$- E_1 S_1 \frac{\partial u_1}{\partial x_1} + \rho_1 \nu_1^2 I_{01} \frac{\partial^3 u_1}{\partial x_1 \partial t^2} + E_2 I_2 \frac{\partial^3 w_2}{\partial x_2^3} - \rho_2 I_2 \frac{\partial^3 w_3}{\partial x_2 \partial t^2} + E_3 S_2 \frac{\partial u_3}{\partial x_3} -$$
$$- \rho_3 \nu_3^2 I_{03} \frac{\partial^3 u_3}{\partial x_3 \partial t^2} - E_4 I_4 \frac{\partial^3 w_4}{\partial x_4^3} + \rho_4 I_4 \frac{\partial^3 w_4}{\partial x_4 \partial t^2} = 0$$

the balance of the transverse and longitudinal forces on the boundary, and

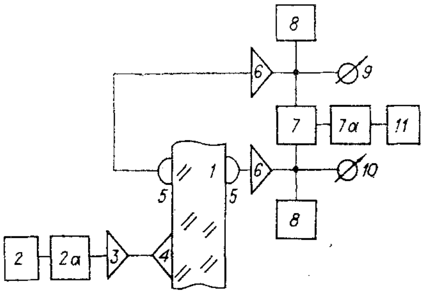

Fig. 3

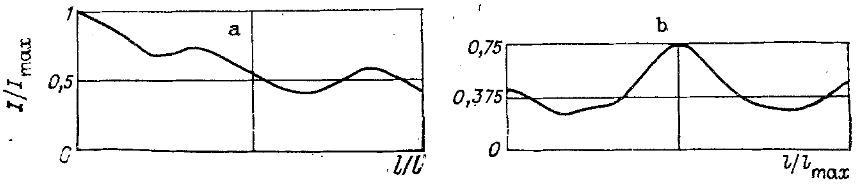

Fig. 4

TABLE 1

Rod number	$I \cdot 100\%/I_{max}$ theory	$I \cdot 100\%/I_{max}$ exper.	Rod number	$I \cdot 100\%/I_{max}$ theory	$I \cdot 100\%/I_{max}$ exp.
1	24.5	30	3	19.5	26
2	28	22.5	4	28	22,5

$$E_1 I_1 \frac{\partial^2 w_1}{\partial x_1^2} + E_2 I_2 \frac{\partial^2 w_2}{\partial x_2^2} + E_3 I_3 \frac{\partial^2 w_3}{\partial x_3^2} + E_4 I_4 \frac{\partial^2 w_4}{\partial x_4^2} = 0$$

the balance of the bending moments on the boundary.

If a bending (or longitudinal) wave incident on the connection is excited in rod 1, then transmitted waves, bending and longitudinal, can occur in rods 2-4 while reflected waves of both kinds can occur in rod 1.

Analytic expressions for the energy reflection and transmission coefficients are obtained by the method of generalized dynamic stiffnesses [2] in each of the connection rods during excitation of a longitudinal or bending wave in rod 1. Because these calculations are cumbersome, we do not present them here. A numerical computation on an ES M4030 was used to analyze the expressions obtained. The computations were performed for organic glass rods of rectangular cross section and area S = 5 × 2.5 cm.

Figure 2 shows the frequency characteristics of the energy reflection and transmission coefficients for the case when a bending wave was excited in rod 1 (G_1 is the energy reflection coefficient in rod 1 in the form of a bending wave, and τ_m, σ_m are the energy transmission coefficients in the m-th rod in the form of bending and longitudinal waves, respectively (m = 2, 3, 4)]. It is easy to see that there is no transformation of the kinds of waves in the 10 Hz-5 kHz frequency band, nor any wave transmission through the connection. The major portion of the incident wave energy is reflected into the first rod in the form of a bending wave. As the frequency grows further the fractions of the transmitted wave energy and the transformations of bending into longitudinal waves increase for the side rods.

2. A correlation method for determining the symmetry of the vibrations was used for experimental investigations of the vibration progapation in rod systems. Figure 3 shows a block diagram of the installation of measure the static characteristics. Here 1 is the structure element, 2 is the noise generator, 3 is a power amplifier, 4 is an emitter, 5 is an accelerometer, 6 is an amplifier, 7 is the correlometer, 8 is an oscilloscope, 9, and 10 are voltmeters; 11 is the recorder, 2a is a set of filters, and 7a is a divider. One of the rods was excited by a piezoceramic plate inscribed by an electrical signal from the noise generator. The mean frequency of the random vibrations spectrum is 10 kHz and its halfwidth is 1 kHz. The piezoceramic plate was located either at the rod endface (mainly longitudinal vibrations were excited in the primary rod), or at its upper face (mainly bending vibrations

were excited). The receiving accelerometers were arranged symmetrically on the upper and and lower faces at different sections of the primary and secondary rods. The mutual correlation function of the signals from both accelerometers was measured by using the correlometer X6-4. For symmetric (longitudinal) vibrations, when the displacements are cophasal at the point of reception, the maximum of the mutual correlation function is positive, while negative for antisymmetric (bending) vibrations.

Investigations showed that longitudinal vibrations are transformed into bending at the during passage of the node and transformation into a longitudinal wave is insignificant.

The autocorrelation function by which the vibration intensity distribution I was judged in the connection elements, was also measured by using the correlometer. The vibration intensity distribution along the length is represented in Fig. 4: a) the excited side of the structure (rods 1 and 3), b) the side not excited (rods 2 and 4). The experimental intensity distribution was compared with that computed theoretically for the frequency 10 kHz (see Table 1). Comparison shows that the theoretical analysis agrees with experiment.

In conclusion, let us note that the theoretical computations in this paper were performed by a linear mathematical model. The longitudinal and bending waves were here propagated independently in the rods and the transformation of the kind of waves only occurred during their transmission through the connections. However, the presence of an elastic nonlinearity in the material can result in interaction between the longitudinal and bending waves on the rectilinear part of the rod [5]. The presence of certain quantitative discrepancies between the theoretical computation and the experiment can be explained by this factor in particular.

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